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FORCED CONVECTION OVER ROTATING BODIES WITH NON-UNIFORM SURFACE TEMPERATURE

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Abstract—An analytical method is developed for obtaining the temperature distribution and the rate of heat transfer in laminar boundary-layer forced convective flow over a rotating body of revolution having a step change in surface temperature. By using a special coordinate transformation and an appropriate series expansion of the temperature, the energy equation becomes expressible in terms of a set of partial differential equations which contain universal functions. These universal functions can be tabulated once and for all. Numerical examples are presented for an isothermal surface and for a surface which has a step discontinuity in temperature for the special cases of a rotating sphere and a rotating disk. These results are compared with values obtained from other formulas available in the literature.

NOMENCLATURE

b, $\left(\dfrac{a_2(\xi)Pr}{3!}\right)^{1/3}$;
B, rotation parameter, $\left(\dfrac{2}{3}\dfrac{R\Omega}{U_\infty}\right)^2$ for sphere, $\dfrac{\pi\Omega R}{2U_\infty}$ for disk;
f, dimensionless stream function, defined in (5a, b);
g, dimensionless rotating velocity, defined in (5c);
L, characteristic length for body;
Nu, Nusselt number;
Pr, Prandtl number $= \nu/\alpha$;
q_w, wall heat flux;
r, radius of body at x;
R, sphere or disk radius;
Re_R, Reynolds number $= \dfrac{RU_\infty}{\nu}$;
T, temperature;
u, velocity component in the x direction;
U_e, velocity at outer edge of boundary layer;
U_∞, approach velocity;
v, velocity component in y direction;
w, velocity component in rotating direction;
x, coordinate measured along surface from front stagnation point;
x_0, location in x direction where surface temperature has a step change;
X, coordinate defined in (9a);
y, coordinate measured normal to x;
z, coordinate measured in rotating direction.

Greek symbols
α, thermal diffusivity;
β, x/R;
η, dimensionless y coordinate, defined in (4);
θ, dimensionless temperature $= \dfrac{T-T_\infty}{T_w-T_\infty}$;
Λ, wedge parameter, defined in (6);
ν, kinematic viscosity;
ξ, dimensionless x coordinate, defined in (3);
ξ_0, dimensionless coordinate defined in (9c);
ζ, dimensionless coordinate defined in (9b);
Ω, angular velocity.

Subscripts
w, evaluated at wall;
∞, evaluated at approach conditions.

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1. INTRODUCTION

BY SPINNING an axisymmetrical body in a forced flow field, the fluid near the surface of the spinning body is forced outward in the radial direction due to the action of centrifugal forces. This fluid is then replaced by fluid moving in the axial direction and, therefore, the axial velocity of a fluid in the neighborhood of a spinning body has a higher value as compared to that for a non-spinning body. This increase in axial velocity results in enhancing the convective heat-transfer rate between the body and the fluid.

The application of the above-mentioned idea in order to develop rotating systems for enhancing the heat-transfer rate has been the subject of many investigations for the past two decades. For example, Hickman [1] envisioned the possibility of rotating condensors for sea water and spacecraft power plants. Ostrach and Braun [2] proposed a method of cooling the nose cones of space vehicles during re-entry by spinning the nose in order to set the surrounding fluid into a rotating motion. By veil cooling of the hot rotating surfaces, Rossler and Mitchell [3] reported that the performance of radial flow gas turbines has been improved. The number of journal articles dealing with flow and heat transfer in rotating systems has also been increasing rapidly.

The interested reader may refer to a monograph by Kreith [4], as only the articles that are closely related to the present investigation will be cited here.

The heat transfer from an isothermal rotating sphere was first analyzed by Siekmann [5] using the Blasius series technique. An exact analysis for the laminar forced flow against a rotating disk having either a uniform surface temperature or a power-law surface temperature distribution was given by Tien and Tsuji [6]. Recently, the authors [7] have analyzed the momentum and heat transfer rates through laminar boundary layers over rotating isothermal bodies of revolution of fairly arbitrary shape by employing Merk's series expansion technique [8]. Numerical results were presented for a sphere and a disk. Chao and Greif [9] have proposed a technique for solving the non-uniform surface temperature case by expanding a two-term velocity profile in a Taylor series in terms of the coordinate normal to the surface and then using a unique coordinate transformation which is similar to that used for two-dimensional stationary bodies [10]. Following this approach, they were able to express the solution in terms of universal functions and as a perturbation from Lighthill's [11] one-term linear velocity profile result.

For a non-rotating body, the Prandtl number alone determines the thickness ratio of the momentum to the thermal boundary layer and, therefore, for large Prandtl numbers or for the region near the point of discontinuity, the two-term velocity profile may accurately represent the actual velocity profile in the regions where significant heat transfer occurs. For a rotating body, however, this thickness ratio depends not only on the Prandtl number but also inversely on the rotating velocity. This implies that for high rotating velocities, the momentum boundary layer–thermal boundary-layer thickness ratio decreases, and, therefore, the velocity profiles in the regions where significant heat transfer occurs cannot be accurately represented by a two-term expression. The error in using the two-term velocity profile in solving the energy boundary-layer equation becomes especially significant for lower Prandtl numbers. In recognizing this difficulty, Chao [12] extended his original technique [10] to take into account more terms in the velocity profile in the solution of the energy boundary-layer equation. In his technique, the temperature profile was expressed as a multi-infinite series and the accuracy of the results depends both on the convergence of the series and the method of series truncation. The accuracy of Chao's results has only been discussed for the case of a rotating disk.

In the present investigation, an entirely different analytical method is proposed. The great advantage of the present method is that one can refine a solution by obtaining more terms in the series solution in a straightforward way, thus taking into account as many higher order terms in the velocity profile as one desires. The two-term velocity profile limitation used in [9] can therefore be relaxed. In addition, our temperature results have less terms in the series for the same order of the velocity profile as compared to Chao's [12] method, and are thereby easier to apply for both uniform and non-uniform surface conditions. Numerical results are presented below for a rotating sphere and a rotating disk.

2. PROBLEM STATEMENT AND ANALYSIS

Consider the steady, laminar axisymmetrical forced convective flow of an incompressible fluid over a rotating body. An initial portion of the rotating body of length x_0 is kept at the same temperature, T_∞, as that of the incoming fluid, and the remaining portion of the rotating body has a surface temperature step change to a uniform value T_w. The physical model and the coordinate system are shown in Fig. 1. The resulting temperature variation is limited so that changes in fluid properties are assumed to be small and thus are neglected, and the velocities involved are not so large that viscous heating is important.

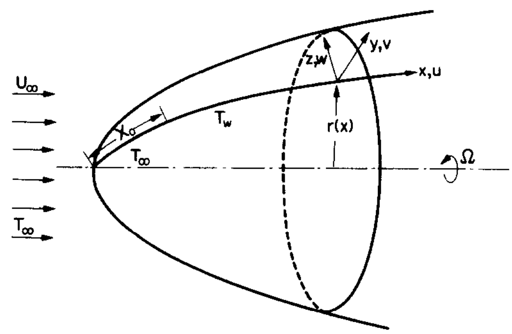

FIG. 1. Physical model and coordinate system.

With the above assumptions, the energy boundary-layer equation is linear and the solution for any arbitrary surface temperature distribution can be obtained by superposition. If one chooses non-rotating coordinates x, y and z, with x representing the distance measured along a meridian curve from the forward stagnation point of the body, y representing the distance normal to the body, and z the distance in the direction of rotation, the boundary-layer energy equation for the problem is

$$u\frac{\partial\theta}{\partial x}+v\frac{\partial\theta}{\partial y}=\alpha\frac{\partial^2\theta}{\partial y^2} \tag{1}$$

with the boundary conditions

$$\begin{aligned}\theta(x,0) &= 1(x-x_0)\\ \theta(x_0,y>0) &= 0\\ \theta(x,\infty) &= 0\end{aligned} \tag{2}$$

where $\theta = (T-T_\infty)/(T_w-T_\infty)$ and $1(x-x_0)$ is the Heaviside unit operator. The general method of obtaining the velocity components u, v and w, has been reported in the previous paper [7], and will be briefly summarized here along with the results.

To solve the momentum equation, the following dimensionless coordinate transformation is introduced, following Merk [14],

$$x \to \xi = \int_0^x \frac{U_e}{U_\infty} \frac{r^2}{L^2} \frac{dx}{L} \tag{3}$$

$$y \to \eta = \left(\frac{Re_L}{2\xi}\right)^{1/2} \frac{U_e}{U_\infty}\left(\frac{r}{L}\right)\left(\frac{y}{L}\right). \tag{4}$$

The velocity components u, v and w which satisfy the equations of continuity and momentum and the boundary conditions are given in [7] as

$$u = U_e \frac{\partial f}{\partial \eta} \tag{5a}$$

$$v = -\frac{r}{L}\frac{U_e}{(2\xi Re_L)^{1/2}} \times \left[f + 2\xi\frac{\partial f}{\partial \xi} + \left(\Lambda + \frac{2\xi}{r}\frac{dr}{d\xi} - 1\right)\eta\frac{\partial f}{\partial \eta}\right] \tag{5b}$$

$$w = r\Omega g(\xi, \eta) \tag{5c}$$

where Λ is a "wedge parameter" defined by

$$\Lambda = \frac{2\xi}{U_e}\frac{dU_e}{d\xi}. \tag{6}$$

The dimensionless stream function $f(\xi,\eta)$ and dimensionless rotating velocity function $g(\xi,\eta)$ satisfy, respectively,

$$f''' + ff' + \Lambda(1 - f'^2) + \left(\frac{2\xi}{r}\frac{dr}{d\xi}\cdot\frac{r^2\Omega^2}{U_e^2}\right)g^2 = 2\xi\left[f'\frac{\partial f'}{\partial \xi} - f''\frac{\partial f}{\partial \xi}\right] \tag{7a}$$

$$g'' + fg' - gf'\left(\frac{4\xi}{r}\frac{dr}{d\xi}\right) = 2\xi\left[f'\frac{\partial g}{\partial \xi} - g'\frac{\partial f}{\partial \xi}\right] \tag{7b}$$

with the boundary conditions

$$f = f' = 0,\ g = 1 \quad \text{for} \quad \eta = 0 \tag{8a}$$

$$f' = 1,\ g = 0 \quad \text{for} \quad \eta \to \infty. \tag{8b}$$

In the foregoing, the primes denote differentiation with respect to η. Although the velocity components u and v do not depend on the function g explicitly, the solution for $f(\xi,\eta)$ must be obtained by a simultaneous solution of (7a) and (7b). The general solution method using Merk's series is proposed in [7], in which numerical results were reported for the rotating sphere and the rotating disk.

The main objective of the present analysis is to obtain the solution of the energy equation (1) satisfying the boundary conditions (2). To this end, we further introduce a transformation [13] as

$$X = \left[1 - \left(\frac{\xi_0}{\xi}\right)^{3/4}\right]^{1/3} \tag{9a}$$

$$\zeta = \frac{b(\xi)\eta}{X} \tag{9b}$$

with

$$\xi_0 = \int_0^{x_0} \frac{U_e}{U}\frac{r^2}{L^2}\frac{dx}{L} \tag{9c}$$

and where $b(\xi)$ is an undetermined parameter to be specified later. The transformations (9a–c) are a generalization of the transformations used by Chao and Cheema [15] in treating forced convection in wedge flow with a step discontinuity in surface temperature. Making use of the velocity components u and v given by (5a,b) and transforming the variables x to X and η to ζ according to (9a,b), the energy equation (1) becomes

$$\frac{\partial^2\theta}{\partial\zeta^2} + \frac{Pr}{b}\left[X\left(f + 2\xi\frac{\partial f}{\partial \xi}\right) + \frac{1}{2b}\frac{(1-X^3)}{X}\zeta\frac{\partial f}{\partial \eta} - \frac{2X^2}{b^2}\xi\frac{db}{d\xi}\zeta\frac{\partial f}{\partial \eta}\right]\frac{\partial \theta}{\partial \zeta} - \frac{Pr}{2b^2}(1-X^3)\frac{\partial f}{\partial \eta}\frac{\partial \theta}{\partial X} = 0 \tag{10}$$

with the boundary conditions

$$\theta(X,\zeta) = 1 \quad \text{for} \quad \zeta = 0 \tag{11a}$$

$$\theta(X,\zeta) = 0 \quad \text{for} \quad \zeta \to \infty. \tag{11b}$$

In (11a,b), $0 \leqslant X \leqslant 1$ and $0 \leqslant \zeta \leqslant \infty$, and the argument η of the velocity function is related to ζ according to (9b). At the location where the temperature has a discontinuity, $\xi = \xi_0$ and $X = 0$, so $\zeta \to \infty$. The entrance condition merges into (11b).

To obtain the solution to (10), we first expand $f(\xi,\eta)$ and $g(\xi,\eta)$ in power series of the form

$$f(\xi,\eta) = \sum_{n=2}^{\infty} \frac{a_n(\xi)}{n!}\eta^n \tag{12}$$

and

$$g(\xi,\eta) = 1 + \sum_{n=1}^{\infty} \frac{b_n(\xi)}{n!}\eta^n. \tag{13}$$

The quantities $(2\xi/r\, dr/d\xi \cdot r^2\Omega^2/U_e^2)$ and $(4\xi/r\, dr/d\xi)$ in (7a,b) are functions of x only, and since x is transformed to ξ, these quantities can be treated as functions of ξ only. The coefficient's a_n's and b_n's in (12) and (13) can therefore be determined by substituting (12) and (13) into (7a) and (7b), respectively, and equating the coefficients of like powers in η. The results are

$$\left.\begin{aligned} a_2 &= \left.\frac{\partial^2 f}{\partial \eta^2}\right|_{\eta=0}, \\ a_3 &= -\Lambda - \frac{2\xi}{r}\frac{dr}{d\xi}\frac{r^2\Omega^2}{U_e^2} \\ a_4 &= -2b_1\frac{2\xi}{r}\frac{dr}{d\xi}\frac{r^2\Omega^2}{U_e^2} \\ a_5 &= a_2^2(2\Lambda - 1) - 2b_1^2\frac{2\xi}{r}\frac{dr}{d\xi}\frac{r^2\Omega^2}{U_e^2} \\ &\quad + 2\xi a_2 a_2', \ldots, \text{etc., and} \end{aligned}\right\} \tag{14}$$

$$\left.\begin{aligned} b_1 &= \left.\frac{\partial g}{\partial \eta}\right|_{\eta=0}, \quad b_2 = 0 \\ b_3 &= a_2\frac{4\xi}{r}\frac{dr}{d\xi}, \ldots, \text{etc.,} \end{aligned}\right\} \tag{15}$$

where primes denote differentiation with respect to ξ.

Substituting $\eta = \zeta X/b$ into (12), the series for the f function becomes

$$f = \frac{a_2}{2!}\frac{\zeta^2}{b^2}X^2 + \frac{a_3}{3!}\frac{\zeta^3}{b^3}X^3 + \frac{a_4}{4!}\frac{\zeta^4}{b^4}X^4 + \frac{a_5}{5!}\frac{\zeta^5}{b^5}X^5 + \dots \quad (16)$$

from which we obtain

$$\frac{\partial f}{\partial \eta} = a_2\frac{\zeta}{b}X + \frac{a_3}{2!}\frac{\zeta^2}{b^2}X^2 + \frac{a_4}{3!}\frac{\zeta^3}{b^3}X^3 + \dots \quad (17)$$

and

$$\frac{\partial f}{\partial \xi} = \frac{a_2'}{2!}\frac{\zeta^2}{b^2}X^2 + \frac{a_3'}{3!}\frac{\zeta^3}{b^3}X^3 + \frac{a_4'}{4!}\frac{\zeta^4}{b^4}X^4 + \frac{a_5'}{5!}\frac{\zeta^5}{b^5}X^5 + \dots . \quad (18)$$

We further seek a series solution for $\theta(\xi, \zeta)$ of the form

$$\theta = \sum_{n=0}^{\infty} \theta_n(\xi, \zeta)X^n \quad (19)$$

with

$$\theta_0(\xi, 0) = 1,\ \theta_1(\xi, 0) = \theta_2(\xi, 0) = \dots = 0 \quad (20a)$$

$$\theta_0(\xi, \infty) = \theta_1(\xi, \infty) = \dots = 0. \quad (20b)$$

By substituting (16), (17), (18) and (19) into (10), and collecting terms of like powers in X, we obtain a sequence of second order, linear differential equations which the θ_n's have to satisfy. These equations (for $n \geq 1$) depend explicitly on the Prandtl number and on the a_n's in the dimensionless velocity function (16), and we therefore rewrite the θ_n's in terms of universal functions so that the solutions can be evaluated once and for all. To this end, let us rewrite $\theta_1, \theta_2, \theta_3$, etc., as

$$\theta_1 = M\bar{\theta}_1 \quad (21)$$

$$\theta_2 = M^2\bar{\theta}_{2,1} + N\bar{\theta}_{2,2} \quad (22)$$

$$\theta_3 = M^3\bar{\theta}_{3,1} + P\bar{\theta}_{3,2} + MN\bar{\theta}_{3,3} + Q\bar{\theta}_{3,4} \quad (23)$$

etc., ..., with

$$M = -\frac{3a_3}{2a_2 b}, \quad N = -\frac{a_4}{2a_2 b^2}$$

$$P = -\frac{a_5}{a_2^2 Pr}, \quad Q = \frac{3\xi a_2'}{a_2}.$$

By using these universal functions and repeating the same operations as described before, we find that θ_0, $\bar{\theta}_1$ and the $\bar{\theta}_{n,j}$'s satisfy the following equations:

$$\theta_0'' + 3\zeta^2\theta_0' = 0 \quad (24)$$

$$\bar{\theta}_1'' + 3\zeta^2\bar{\theta}_1' - 3\zeta\bar{\theta}_1 = \theta_0' \quad (25)$$

$$\bar{\theta}_{2,1}'' + 3\zeta^2\bar{\theta}_{2,1}' - 6\zeta\bar{\theta}_{2,1} = \zeta^3\bar{\theta}_1' - \zeta^2\bar{\theta}_1 \quad (26a)$$

$$\bar{\theta}_{2,2}'' + 3\zeta^2\bar{\theta}_{2,2} - 6\zeta\bar{\theta}_{2,2} = \zeta^4\theta_0' \quad (26b)$$

$$\bar{\theta}_{3,1}'' + 3\zeta^2\bar{\theta}_{3,1}' - 9\zeta\bar{\theta}_{3,1} = \zeta^3\bar{\theta}_{2,1} - 2\zeta^2\bar{\theta}_{2,1} \quad (27a)$$

$$\bar{\theta}_{3,2}'' + 3\zeta^2\bar{\theta}_{3,2}' - 9\zeta\bar{\theta}_{3,2} = \tfrac{3}{4}\zeta^5\theta_0' \quad (27b)$$

$$\bar{\theta}_{3,3}'' + 3\zeta^2\bar{\theta}_{3,3}' - 9\zeta\bar{\theta}_{3,3} = \zeta^4\bar{\theta}_1' - \zeta^3\bar{\theta}_1 + \zeta^3\bar{\theta}_{2,2}' - 2\zeta^2\bar{\theta}_{2,2} \quad (27c)$$

$$\bar{\theta}_{3,4}'' + 3\zeta^2\bar{\theta}_{3,4}' - 9\zeta\bar{\theta}_{3,4} = (-3\zeta^2 + 2\zeta)\theta_0' \quad (27d)$$

with the associated boundary conditions

$$\theta_0(\xi, 0) = 1,\ \bar{\theta}_1(\xi, 0) = \bar{\theta}_{2,1}(\xi, 0) = \dots = \bar{\theta}_{3,4}(\xi, 0) = 0$$

$$\theta_0(\xi, \infty) = \bar{\theta}_1(\xi, \infty) = \bar{\theta}_{2,1}(\xi, \infty) = \dots = \bar{\theta}_{3,4}(\xi, \infty) = 0.$$

In the above equations (24)–(27d), we have defined the undetermined parameter $b(\xi)$ as $b(\xi) = (a_2 Pr/3!)^{1/3}$, and this value has been substituted into the LHS of the equations. The primes appearing in these equations denote differentiation with respect to ζ. By choosing the parameter $b(\xi)$ in this form, we are able to express equations (24)–(27d) in forms that are independent of the Prandtl number and Λ, and they can therefore be solved once and for all. For wedge flow (non-rotating), a_2 becomes constant and reduces to the form defined in [15].

Solutions for equations (24) and (25) can be obtained in closed form and are

$$\theta_0(\zeta) = 1 - \frac{\Gamma(1/3, \zeta^3)}{\Gamma(1/3)} \quad (28)$$

$$\bar{\theta}_1(\zeta) = \frac{1}{5\Gamma(1/3)}\zeta[\Gamma(4/3) - \Gamma(4/3, \zeta^3)]. \quad (29)$$

Their derivatives at the surface are, respectively,

$$\theta_0'(0) = -\frac{3}{\Gamma(1/3)} = -1.1198 \quad (30)$$

and

$$\bar{\theta}_1'(0) = 1/15. \quad (31)$$

Equations (26a), (27a) and (27b) are precisely identical to those given in [15] for wedge flow with a non-rotating body (in [15] the notation is $\bar{F}_2 = \bar{\theta}_{2,1}$, $\bar{F}_{3,1} = \bar{\theta}_{3,1}$, and $\bar{F}_{3,2} = \bar{\theta}_{3,2}$). These equations are numerically integrated in [15] and the universal functions tabulated in the paper can therefore be directly applied to the present problem. The remaining equations, (26b), (27c) and (27d) were numerically integrated using a fourth order Runge–Kutta procedure with a uniform step size of $\Delta\zeta = 0.01$. The resulting values of $\bar{\theta}_{2,2}$, $\bar{\theta}_{3,3}$ and $\bar{\theta}_{3,4}$ are tabulated in Table 1. To calculate the surface heat flux, the associated surface derivatives are required, and they are tabulated for the reader's convenience as follows:

$$\theta_0'(0) = -1.1198;\ \bar{\theta}_1'(0) = 1/15;$$
$$\bar{\theta}_{2,1}'(0) = 0.81748 \times 10^{-2};\ \bar{\theta}_{2,2}'(0) = 0.40871 \times 10^{-1};$$
$$\bar{\theta}_{3,1}'(0) = 0.17204 \times 10^{-2};\ \bar{\theta}_{3,2}(0) = 0.20737 \times 10^{-1};$$
$$\bar{\theta}_{3,3}'(0) = 0.12903 \times 10^{-1};\ \bar{\theta}_{3,4}'(0) = 0.64167 \times 10^{-1}.$$

With the availability of these universal functions and their surface derivatives, the determination of the temperature field in the boundary layer and the local heat-transfer rate for any arbitrary Prandtl number and rotating parameter reduces to a simple algebraic operation. For convenience, the dimensionless temperature and local wall heat flux can be

Table 1. Universal functions $\bar{\theta}_{n,j}$

ζ	$\bar{\theta}_{2,2}\times 10$	$\bar{\theta}_{3,3}\times 10$	$\bar{\theta}_{3,4}\times 10$
0.0	0	0	0
0.1	0.04088	0.01291	0.06075
0.2	0.08189	0.02590	0.10351
0.3	0.12318	0.03918	0.11732
0.4	0.16462	0.05305	0.09902
0.5	0.20522	0.06781	0.05279
0.6	0.24271	0.08395	−0.01118
0.7	0.27341	0.10141	−0.07917
0.8	0.29274	0.11982	−0.13729
0.9	0.29646	0.13781	−0.17487
1.0	0.28227	0.15293	−0.18704
1.1	0.25108	0.16183	−0.17544
1.2	0.20732	0.16131	−0.14691
1.3	0.15795	0.14969	−0.11066
1.4	0.11037	0.12795	−0.07516
1.5	0.07032	0.09980	−0.04602
1.6	0.04063	0.07044	−0.02535
1.7	0.02116	0.04467	−0.01252
1.8	0.00988	0.02528	−0.00553
1.9	0.00411	0.01269	−0.00217
2.0	0.00152	0.00561	−0.00076
2.1	0.00049	0.00218	−0.00023
2.2	0.00014	0.00074	−0.00006
2.3	0.00004	0.00021	−0.00001

The universal functions for $\bar{\theta}_{2,1}$, $\bar{\theta}_{3,1}$ and $\bar{\theta}_{3,2}$ can be found in Table 1 of [15] in which these functions are denoted, respectively, by $\bar{F}_2$, $\bar{F}_{3,1}$ and $\bar{F}_{3,2}$.

recast in terms of these universal functions and their derivatives as

$$\theta(\xi,\zeta,X) = \sum_{n=0}^{\infty}\theta_n(\xi,\zeta)X^n$$
$$= \theta_0 + M\bar{\theta}_1 X + (M^2\bar{\theta}_{2,1} + N\bar{\theta}_{2,2})X^2 + (M^3\bar{\theta}_{3,1} + P\bar{\theta}_{3,2} + MN\bar{\theta}_{3,3} + Q\bar{\theta}_{3,4})X^3 + \dots \quad (32)$$

and

$$q_w = -k\left(\frac{\partial T}{\partial y}\right)_{y=0}$$
$$= 2^{-1/2}(3!)^{-1/3}k(T_w - T_\infty)\left(\frac{Re_L}{\xi}\right)^{1/2} \times \left(\frac{U_e}{U_\infty}\right)\frac{r}{L^2}X^{-1}(Pra_2)^{1/3}\left(-\frac{\partial\theta}{\partial\zeta}\right)\bigg|_{\zeta=0} \quad (33a)$$

where

$$\left(-\frac{\partial\theta}{\partial\zeta}\right)\bigg|_{\zeta=0} = \sum_{n=0}^{\infty}\theta_n'(\xi,0)X^n$$
$$= 1.11985 - M/15X - (0.81748\times 10^{-2}M^2 + 0.40872\times 10^{-1}N)X^2 - (0.17204\times 10^{-2}M^3 + 0.20737\times 10^{-1}P + 0.12903\times 10^{-1}MN + 0.64167\times 10^{-1}Q)X^3 + \dots. \quad (33b)$$

As has been demonstrated in the text, the derivation of the above equations does not impose any approximations or limitations, and therefore they are an exact form of series expression. With straightforward numerical integration of the higher order terms of the θ_n's in (19), we may readily provide more terms in the series solution (32) and (33a,b) such that the higher order terms in the velocity distribution (12) can be properly accounted for in the solution to the energy equation. The accuracy of the results by using a finite number of terms in the series provided in this paper depends largely on the convergence of the series, i.e. on the Prandtl number, rotating velocity and the range of X. For small Prandtl number and large rotating velocity and $X \sim 1$, the series may become semi-divergent and Euler's summation method [16] may be used for summing the series. For large Prandtl number, or in the region close to the point of surface temperature discontinuity, the thermal boundary-layer thickness is comparatively much thinner than that of the velocity boundary layer so that a linear velocity distribution may be used for the solution of the energy equation as done by Lighthill [11]. This is the case of retaining the first term in (16), i.e. $f = (a_2/2)(\zeta^2/b^2)X^2$. By substituting this velocity profile into (10) and performing a similar manipulation as before, the resulting temperature field is

$$\theta_l = \theta_0 + Q\bar{\theta}_{3,4}X^3 + \dots \quad (34)$$

where the subscript l denotes the solution for a linear velocity profile. The corresponding wall heat flux is

$$q_{w_l} = 2^{-1/2}(3!)^{-1/3}k(T_w - T_\infty) \times \left(\frac{Re_L}{\xi}\right)^{1/2}\left(\frac{U_e}{U_\infty}\right)\frac{r}{L^2}X^{-1}(Pra_2)^{1/3} \times [1.11985 - 0.64167\times 10^{-1}QX^3]. \quad (35)$$

These two expressions can also be obtained from (32) and (33a,b) by simply letting $Pr \to \infty$. By comparing equations (34) and (35) with (32) and (33a,b), the excess terms appearing in the latter equations are obviously due to the contribution of the second and third terms of the velocity distribution in (16). The application of the present method to the cases of a rotating sphere and a rotating disk will be made in the next section.

3. APPLICATION TO A ROTATING SPHERE

In this section, we would like to demonstrate how one can apply the general equation (33a,b) to calculate the heat transfer characteristics of a rotating sphere. For a spherical body, it is known from potential flow theory that $U_e/U_\infty = 3/2\sin X/R$, and from the geometry, $r/R = \sin x/R$. With U_e/U_∞ and r/R thus defined, the quantities $(2\xi/r\,\mathrm{d}r/\mathrm{d}\xi\cdot r^2\Omega^2/U_e^2)$ and $(4\xi/r\,\mathrm{d}r/\mathrm{d}\xi)$ appearing in (14) and (15) become, respectively,

$$\frac{2\xi}{r}\frac{\mathrm{d}r}{\mathrm{d}\xi}\frac{r^2\Omega^2}{U_e^2} = B\Lambda \quad (36)$$

$$\frac{4\xi}{r}\frac{\mathrm{d}r}{\mathrm{d}\xi} = 2\Lambda \quad (37)$$

where

$$B = \left(\frac{2}{3}\frac{R\Omega}{U_\infty}\right)^2$$

$$\xi = 1-1.5\cos\beta+0.5\cos^3\beta$$

$$\Lambda = \frac{4\cos\beta-6\cos^2\beta+2\cos^4\beta}{3\sin^4\beta}$$

and

$$\beta = \frac{x}{R}.$$

Defining a local Nusselt number as $Nu = Rq_w/k(T_w - T_\infty)$ and replacing the characteristic length L by R, we obtain

$$NuRe_R^{-1/2} = 0.38914(Pra_2)^{1/3} \times \frac{U_e}{U_\infty}\frac{r}{R}X^{-1}\xi^{-1/2}\left(-\frac{\partial\theta}{\partial\zeta}\right)_{\zeta=0} \quad (38a)$$

in which $(-\partial\theta/\partial\zeta)_{\zeta=0}$ is given by (33b) but with

$$M = \frac{3\Lambda(1+B)}{2a_2 b}, \quad N = \frac{\Lambda B b_1}{a_2 b^2}$$

$$P = \frac{1}{Pr}\left[(1-2\Lambda)+\frac{2\Lambda B b_1^2}{a_2^2}-\frac{2\xi a_2'}{a_2}\right], \quad (38b)$$

$$Q = \frac{2\xi a_2'}{a_2}.$$

The values of $a_2(\xi)$ and $b_1(\xi)$ in (38a,b) can be obtained from the solution of the momentum boundary-layer equations in the following Merk's type of series expressions refined in [8] as

$$a_2(\xi) = f''(\Lambda,0) = f_0''(\Lambda,0)+2\xi\frac{d\Lambda}{d\xi}f_1''(\Lambda,0) + 4\xi^2\frac{d^2\Lambda}{d\xi^2}f_2''(\Lambda,0)+\ldots \quad (39)$$

$$b_1(\xi) = g'(\Lambda,0) = g_0'(\Lambda,0)+2\xi\frac{d\Lambda}{d\xi}g_1'(\Lambda,0) + 4\xi^2\frac{d^2\Lambda}{d\xi^2}g_2'(\Lambda,0)+\ldots \quad (40)$$

and the values of $f_i''(\Lambda,0)$ and $g_i'(\Lambda,0)$ are tabulated for $i = 0$, 1 and 2 for various values of the rotation parameter in [7].

To examine the accuracy of the present formula, we first consider the case of an isothermal surface. Under this condition $X = 1$ and the error resulting from using a finite number of terms in the sequence (33b) is the largest due to the slower convergence for larger values of X. The local heat-transfer results, expressed as $NuRe_R^{-1/2}$ for $Pr = 1.0$ are summarized in Table 2. Two sets of our data are presented in the table. The first set of data were obtained by using the values of a_2 and b_1 calculated from the three term Merk's series (39) and (40) using the information given in [7], with the value of a_2' obtained by Taylor's non-uniform interpolation method. The

Table 2. Comparison of $NuRe_R^{-1/2}$ calculated from various methods for an isothermal rotating sphere in forced flow; $Pr = 1$

	$B = 1$					$B = 4$			
	Present analysis					Present analysis			
x/R	Equation (38a)	Equation (38a) with $a_2 = f_0''(\Lambda,0)$ $b_1 = g_0'(\Lambda,0)$ $a_2' = 0$	Merk's series 3 terms [7]	2-term velocity profile [9]	3-term velocity profile [12]	Equation (38a)	Equation (38a) with $a_2 = f_0''(\Lambda,0)$ $b_1 = g_0'(\Lambda,0)$ $a_2' = 0$	Merk's series 3 terms [7]	Siekman [5]
0.0	0.9493	0.9493	0.9588	0.8904	0.9589	1.0007	1.0007	1.0214	1.0214
0.244	0.9390	0.9389	0.9482	0.8815		0.9898	0.9897	1.0099	1.0101
0.474	0.9116	0.9110	0.9195	0.8570		0.9608	0.9602	0.9789	0.9788
0.717	0.8635	0.8620	0.8688	0.8143		0.9101	0.9082	0.9239	0.9237
0.951	0.7991	0.7955	0.7998	0.7567	0.7792*	0.8422	0.8376	0.8484	0.8485
1.103	0.7482	0.7419	0.7436	0.7104		0.7885	0.7804	0.7862	0.7875
1.215	0.7063	0.6968	0.6961	0.6716	0.7064*	0.7445	0.7319	0.7328	0.7362
1.303	0.6495*	0.6575	0.6544	0.6378		0.6888*	0.6892	0.6851	0.6918
1.374	0.6181*	0.6223	0.6171	0.6078	0.6275*	0.6544*	0.6504	0,6414	0.6528
	$B = 10$								
0.0	1.0670	1.0670	1.1141	0.6933	1.0914*				
0.244	1.0555	1.0553	1.1014	0.6885					
0.474	1.0248	1.0239	1.0676	0.6755					
0.717	0.9709	0.9685	1.0061	0.6534					
0.951	0.8991	0.8930	0.9218	0.6245	0.9264*				
1.103	0.9426	0.8315	0.8516	0.6012					
1.215	0.7968	0.7790	0.7904	0.5809	0.8042*				
1.303	0.7535*	0.7323	0.7349	0.5645					
1.374	0.7144*	0.6892	0.6826	0.5440	0.7223*				

*Euler's summation technique was used.

second set were obtained by using the local similarity solution, i.e. one term in the series $[a_2 = f_0''(\Lambda, 0)$ and $b_1 = g_0'(\Lambda, 0)]$. By comparing these two sets of results, the maximum discrepancy is about 3.5% for the range of parameters studied. It is therefore recommended that for engineering applications in which extreme accuracy is not strictly essential, that the local similarity values of a_2 and b_1 may be used for calculation of the heat-transfer characteristics at the surface. By comparing our data with that obtained by the three term Merk's series [7] and the four term Blasius series solution of Siekmann [5] (only the case of $B = 4$ was given in his paper), the agreement is considered to be satisfactory. The results obtained from the formula using the two term velocity profile of [9] generally underestimate the values, especially for the case of $B = 10$. This suggests that under this condition the use of the quadratic velocity profile in the analysis of the energy equation is not adequate. Also included in Table 3 are the Nusselt numbers calculated by the series solution [12] in terms of curvature parameters ε_n's by retaining a three term velocity distribution. Twelve terms in the series, equation (22b) of [12], were used in the calculation. A significant improvement is seen for both $B = 1$ and 10.

Table 3. Heat-transfer rate, $NuRe_R^{-1/2}$, for a rotating sphere with $Pr = 1$

Step-change position	x/R	$NuRe_R^{-1/2}$ $B = 1$	$B = 4$	$B = 10$
$x_0/R = 0.22$	0.244	1.5415	1.6644	1.8445
	0.4739	0.9514	1.0049	1.0765
	0.7165	0.8746	0.9224	0.9853
	0.9507	0.8039	0.8474	0.9052
	1.1030	0.7512	0.7918	0.8465
	1.2148	0.7086	0.7480	0.7997
	1.3027	0.6512*	0.6907*	0.7556*
	1.3744	0.6195*	0.6559*	0.7161*
$x_0/R = 0.90$	0.9507	1.6866	1.8333	2.0493
	1.1030	1.0544	1.1269	1.2345
	1.2148	0.9011	0.9582	1.0429
	1.3027	0.7846*	0.8386*	0.9248*
	1.3744	0.7267*	0.7739*	0.8503*

*Euler's summation technique was used.

The heat-transfer rate for a non-uniform surface temperature can also be readily obtained from (38a,b). The parameter ξ_0 in the transformed variable X now becomes

$$\xi_0 = 1 - \tfrac{3}{2} - \cos\left(\frac{x_0}{R}\right) + \tfrac{1}{2}\cos\left(\frac{x_0}{R}\right).$$

The heat-transfer rates, expressible as $NuRe_R^{-1/2}$, for non-uniform surface temperatures for the special cases of $x_0/R = 0.22$ and $x_0/R = 0.90$ are tabulated in Table 3 for $B = 1$, 4 and 10. To our knowledge there is no previously published data available for comparison with our results.

4. APPLICATION TO A ROTATING DISK

The second numerical example considered is the heat transfer from a finite rotating disk of radius R, with or without a free stream velocity U_∞ impinging normally onto the disk surface. Under this condition, $r = x$, $U_e/U_\infty = 2x/\pi R$, $B = (\pi R\Omega/2U_\infty)$ and, therefore, $\Lambda = 0.5$ which is a constant. The a_2 and b_1 now become $a_2 = [f_0''(\Lambda, 0)]_{\Lambda=0.5}$ and $b_1 = [g_0'(\Lambda, 0)]_{\Lambda=0.5}$ as given by (39) and (40). Since a_2 is independent of ξ, $a_2' = 0$.

Let us define the Nusselt number for this problem in the same manner as that done by Tien and Tsuji [6]. Then (33a,b) can be recast in the following form as

$$\begin{aligned} Nu &= \frac{q_w \nu^{1/2}}{k(T_w - T_\infty)(C^2+\Omega^2)^{1/4}} \\ &= \left(\frac{Pra^*}{3}\right)^{1/3} \frac{1}{X}[1.11985 - 0.2\varepsilon X \\ &\quad -(0.07357 + 0.2312 S_1)\varepsilon^2 X^2 \\ &\quad -(0.04645 + 0.21897 S_1 \\ &\quad + 0.1106 S_2)\varepsilon^3 X^3 + \ldots] \end{aligned} \tag{41}$$

where

$$C = \frac{2U_\infty}{\pi R};\ X = \left[1 - \left(\frac{x_0}{x}\right)^3\right]^{-1/3};\ a^* = \frac{(2)^{1/2}}{(1+B)^{3/4}} a_2$$

$$S_1 = \frac{Ba^* b_1}{[1+B]^{5/4}};\ S_2 = \frac{B(a^* b_1)^2}{[1+B]^{3/2}};$$

$$\varepsilon = \tfrac{3}{2}^{1/3}(a^{*4} Pr)^{-1/3}.$$

The numerical constant a^* depends on the rotation parameter, B, and has been calculated by Hannah [17] ($a^* = 2a$ in her paper). It may also be obtained by the above relation using the value of $a_2 = [f_0''(\Lambda, 0)]_{\Lambda=0.5}$ reported by the authors [7]. The value of b_1 can be shown to be related to b of [17] by the relationship

$$b_1 = [g_0'(\Lambda, 0)]_{\Lambda=0.5} = -\frac{(1+B)^{1/4}}{(2)^{1/2}} b \tag{42}$$

and can be found in [7] or by using b from [17].

For a disk at a uniform temperature, $x_0 = 0$, and we simply substitute $X = 1$ in (41). By a completely different technique, Chao and Greif [9] have obtained an expression for an isothermal rotating disk. In the present notation, it is

$$\begin{aligned} Nu = \left(\frac{Pra^*}{3}\right)^{1/3} &[1.11985 - 0.18868\varepsilon \\ &-0.07271\varepsilon^2 - 0.05079\varepsilon^3 - \ldots]. \end{aligned} \tag{43}$$

By comparing (41) with $X = 1$ with (43), it is seen that the functional forms of both equations are quite similar. The first term on the RHS of both equations is identical, and represents the contribution of the linear component of the velocity profile. The second terms are almost identical and differ only slightly in the value of the numerical coefficient, as do the first elements of the third and fourth terms in (41) when compared with the third and fourth terms of (43). These terms are contributed by the quadratic component of the velocity profile. The elements of the third and fourth terms in (41) which contain S_1 and S_2 are contributed by the third and fourth order terms of the velocity profile, and these elements are naturally missing in (43) since only the quadratic velocity profile was used in its derivation. For a non-rotating body, $B = 0$, and both S_1 and S_2 are equal to zero. In this case, for moderate Prandtl numbers, the two-term velocity profile is adequate for obtaining a solution to the energy equation. For $B \to \infty$, i.e. for pure rotation, we get $S_1 = -0.314$ and $S_2 = 0.0493$ by using the values of a and b given in [17]. The S_1 and S_2 are of the same order of magnitude as the numerical coefficient in the same term, and, for moderate Prandtl numbers but large rotation parameter values, it is thus seen that the two-term velocity profile is not adequate in obtaining the solution to the energy equation. Using a three-term velocity profile, Chao [12]* reported the Nusselt number in the present notation as

$$Nu = \left(\frac{Pra^*}{3}\right)^{1/3}[1.11985-0.18868\varepsilon - [0.072714+0.23647S_1]\varepsilon^2+\ldots]. \quad (44)$$

Surprisingly, the functional form of (44) is quite similar to our equation with a slight difference in the numerical coefficient in the second and third terms. A numerical comparison of the results given by (41), (43) and (44), along with those of Tien and Tsuji [6] obtained by a numerical integration technique, is presented in Table 4. For the case of $B = 4$ and $Pr = 1$, our results deviate by -0.2% from those of [6]. This degree of accuracy is also evident for $B \to \infty$ and $Pr = 1$. For $Pr = 1$ and $B = 4$, the two-term velocity formula (43) deviates by -13.4%. Significant improvement is seen when the three-term velocity formula (44) is used.

Table 4. Comparison of Nusselt number for forced flow against a rotating disk with uniform surface temperature

		Nu			
Pr	*B*	Present analysis equation (41) $X = 1$	Three-term velocity representation [12]	Two-term velocity representation [9]	Tien and Tsuji [6]
1	0	0.755	0.7643	0.7643	0.762
	1	0.652	0.659	0.628	0.658
	4	0.546	0.548	0.484	0.557
	∞	0.3961*	0.400	0.309	0.396
10	0	1.748		1.752	1.752
	1	1.529		1.518	1.535
	4	1.330		1.297	1.340
	∞	1.139		1.059	1.134

*Euler's summation technique was used.

*The ε_1 and ε_2 defined in [12] relate to ε and S_1 as $\varepsilon_1 = -\varepsilon$, $\varepsilon_2 = -4(2)^{1/2}/3\, S_1\varepsilon^2$. The positive sign appearing in front of the second and third terms of (28) in [12] should be negative.

5. CONCLUSIONS

A new formula is presented for calculating the rate of heat-transfer from a rotating body, having a step change in surface temperature, placed in a forced flow stream. A great advantage of the present method is that one can refine the solution by obtaining more terms in the series in a straightforward manner, thus taking into account more terms in the velocity profile. The accuracy of using a finite number of terms in the series solution is discussed for the cases of a rotating sphere and a rotating disk having uniform surface temperatures. For engineering calculations, it is recommended that the similarity solution value, $f_0''(0)$ and $g_0'(0)$, may be used for a_2 and b_1 in the calculation using our formula without introducing a significant error for the rotating sphere for Prandtl numbers equal to or larger than 1. Our formula may also be used for mass-transfer problems by simply replacing θ by $(C-C_\infty)/(C_w-C_\infty)$, and the results may be extended for any arbitrary surface temperature or concentration distribution by the use of superposition. Finally, in order to take into account a predetermined number of terms in the velocity profile in the solution of the energy equation, our formula provides probably the most simple and rapid calculation procedure for the temperature field and the local heat flux for non-isothermal surface conditions as any yet presented in the literature.

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CONVECTION FORCEE SUR DES CORPS TOURNANTS AVEC TEMPERATURE PARIETALE NON UNIFORME

Résumé – On développe une méthode analytique pour calculer la distribution de température et le flux de chaleur en convection forcée avec couche limite laminaire sur un corps de révolution en rotation et ayant un changement de température pariétale en échelon. En utilisant une transformation spéciale des coordonnées et un développement en série approprié, l'équation d'énergie s'exprime en un système d'équations aux dérivées partielles qui contient des fonctions universelles. Ces fonctions peuvent être tabulées une fois pour toutes. On présente des exemples numériques pour une surface isotherme et pour une surface ayant une discontinuité de température en échelon, dans les cas particuliers d'une sphère et d'un disque en rotation. Ces résultats sont comparés avec des valeurs obtenues à partir d'autres formules disponibles dans la bibliographie.

ERZWUNGENE KONVEKTION AN ROTIERENDEN KÖRPERN MIT UNGLEICHFÖRMIGER VERTEILUNG DER OBERFLÄCHENTEMPERATUR

Zusammenfassung—Ein analytisches Verfahren wird entwickelt, um die Temperaturverteilung und die Wärmestromdichte zu ermitteln, die sich in einer erzwungenen Konvektionsströmung mit laminarer Grenzschicht an einem rotierenden Umdrehungskörper ergibt. Die Oberflächentemperatur wird dabei sprungförmig verändert. Durch Verwendung einer speziellen Koordinaten–Transformation und einer geeigneten Reihenentwicklung für die Temperatur läßt sich die Energiegleichung durch ein System partieller Differentialgleichungen ausdrücken, die universelle Funktionen enthälten. Es ist möglich, diese universellen Funktionen ein für allemal zu tabellieren. Es werden numerische Beispiele anhand der Spezialfälle einer rotierenden Kugel und einer rotierenden Scheibe gezeigt, und zwar jeweils für eine isotherme Oberfläche und für eine Oberfläche, die eine sprungförmige Temperaturänderung aufweist. Diese Ergebnisse werden mit Werten verglichen, die mit Hilfe von anderen Gleichungen aus der Literatur gewonnen worden sind.

ВЫНУЖДЕННАЯ КОНВЕКЦИЯ ОКОЛО ВРАЩАЮЩИХСЯ ТЕЛ ПРИ НЕОДНОРОДНОЙ ТЕМПЕРАТУРЕ ПОВЕРХНОСТИ

Аннотация — Разработан аналитический метод определения поля температур и интенсивности теплообмена в ламинарном пограничном слое при вынужденной конвекции около движущегося тела вращения при ступенчатом изменении температуры поверхности. С помощью специального преобразования координат и соответствующего разложения значений температуры уравнение энергии приводится к системе уравнений в частных производных, содержащих универсальные функции, которые можно затабулировать раз и навсегда. Численные примеры приведены для изотермической поверхности и для поверхности, температура которой является ступенчато-разрывной функцией, в случаях вращающейся сферы и вращающегося диска. Полученные результаты сопоставлены с имеющимися в литературе расчётными данными.